Imperfect Labor Contracts and International Trade

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Abstract

In an economy with imperfect labor contracts, differences in the distribution of human capital are an independent source of comparative advantage. I study a world economy with two sectors, one where output is produced by teams and another where individuals can work alone. When workers' abilities are private information and workers cannot verify the value of their output or the level of profits, feasible labor contracts fail to generate efficient matching of workers within teams. The general equilibrium has the least talented individuals entering the team sector, while the most talented opt to work alone. The inefficiencies are more severe in the country with the more heterogeneous labor force, which causes this country to specialize relatively in the good produced by individuals. Trade exacerbates the "polarization" of the more diverse society. National income could be raised and the distribution of income improved, by a marginal expansion in the size of the team sector.

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1 Introduction

Workers differ in ability. Although some like Michael Jordan and Luciano Pavarotti have special talents that are useful only in certain occupations, much of the variation in ability is more general in nature. Individuals who are endowed with high intelligence, good health, and ample energy, and those who have had the benefit of a supportive upbringing and a quality education are potentially more productive in a wide range of activities than those who have been less fortunate along some or all of these dimensions. One of the most important functions of the labor market is to allocate the heterogenous pool of talent to the different sectors of the economy.

In a world of perfect labor contracts (and competitive firms, complete markets, etc.), the allocation of talent would be efficient. A worker of given talents would confront a range of options for different jobs in different sectors and would choose the one that appealed the most. But the 'invisible hand' would guide these choices. Not only would individuals' comparative advantages be reflected in their choices of profession and industry, but the most generally talented individuals would undertake the jobs with the greatest social return to talent, and individuals of similar ability would work together whenever complementarities in the production technology dictated the efficacy of their doing so.

Alas, real world labor contracts are rarely perfect. Imperfections arise from informational asymmetries and the costliness of verifying the contingencies that might appear in a contract. Workers often have better information than prospective employers about the factors that determine their own productivity. When prospective employers do not observe all of the relevant aspects of an applicant's ability, an offer cannot be made fully contingent on talent. A firm might wish, then, to link an employee's compensation to his or her output. But there are at least two potential problems with this. First, an individual's output may be difficult to measure, because the technology may require joint inputs from a number of workers. Then a contract could tie payments only to the output of the team. Second, even this more limited class of contracts may be restricted, if workers cannot readily observe a firm's output or its profits. Piecework and profit-sharing arrangements break down when workers cannot verify an employer's claims about joint production or profits. Firms may be left with little choice but to pay similar compensation to workers whose abilities differ in unobservable ways.

If labor contracts cannot finely distinguish between workers, the allocation of talent may be distorted. To break even, a firm can pay only a wage commensurate with the average productivity of its work force. Such an offer induces adverse selection. A uniform contract that suits the average worker will not appeal to those who know themselves to be more productive than average and who perceive alternative options that would provide greater returns to their talent. Firms that are forced to offer uniform contracts will draw disproportionately from the bottom end of the target population of workers (i.e., those with the observable attributes it demands), while the cream of any group of outwardly similar workers will seek activities in which their output can be measured or where they themselves retain the property rights to the fruits of their labor.

Imperfect employment contracting affects both occupational choice and industry allocation. A talented individual will eschew team activities in which individual attribution is difficult and verification of group output is costly. Within an industry, such an individual might prefer specialties that permit measurement of his personal contribution, or, as in the model presented below, occupations that make him the residual claimant on the output produced by a team. And since industries differ in their technologies, the problems posed by imperfect contracting may be more severe in some sectors than in others. In particular, large-scale manufacturing may be at a disadvantage in attracting the most talented workers as compared to, say, the software, financial or legal sectors, where it is efficient for individuals to work in small groups or even alone.

In a world of imperfect labor contracts, national differences in the distribution of talent can be an independent source of comparative advantage. Two countries that are otherwise identical may specialize in different activities in a competitive, free-trade equilibrium, if talent diversity is greater in one than the other. Consider, for example, the United States and Japan. It is commonly observed that Japan has a more homogeneous labor force than the United States. Suppose the average ability of workers in both countries is the same, and that both countries have access to the same production technologies. Let there be two sectors, one (automobiles) in which team production is required and a second (software) in which workers toil alone. In Section 2, I show that there will be no trade between these countries if employment contracts can be written that make workers' pay contingent on the productivity of their team. In other words, differences in the distribution of talent do not generate comparative advantage when perfect contracts are feasible.[1] But suppose that output is not verifiable, hence contracts cannot link pay to productivity. In the United States, a moderately talented individual might be disinclined to join in team production, because average productivity would be dragged down by those with very low ability. In Japan, the same forces are present, but to a lesser extent. An individual with the same moderate talents might be willing to work in a car plant, if the (average) wage paid to all workers in the sector were not too low. In Section 4, I show that, with trade, a given worker of high talent has a greater incentive to work in the software industry in a country where the distribution of ability is more spread. This leads to the prediction that the country with greater diversity will export software and import cars.

There are some important consequences of the trade that derives from differences in distributions of talent in the face of imperfect labor contracting. First, such trade causes a deterioration of the income distribution in the country with a wider spread of talents. On the margin, an increase in the relative price of software induces the most talented workers in team production under autarky to leave that sector and work by themselves in the export sector. This degrades the talent pool among those

[1]Grossman and Maggi (1998) show that differences in diversity can be a source of international trade even with perfect information or perfect contracts if, for technological reasons, the talents of team workers are are substitutable in some sectors and complementary in others. This is further discussed in Section 5 below.

remaining in the import-competing sector, which depresses average productivity and wages there. It is commonly believed (see, for example, Krugman, 1995) that only the expanded trade with poor, labor-abundant countries can possibly have contributed to the adverse trends in the wages of the unskilled in the United States in the last two decades. My model suggests that, by furthering the incentives for segregation of workers by skill, growing trade with industrialized countries like Japan may also have been responsible for part of the observed trends.

Second, trade associated with imperfect labor contracting can exacerbate a pre-existing production distortion in the country with the more diverse population. A talented individual choosing between the automobile and software industries does not take into account that his employment would generate external benefits in the former sector, but not in the latter. If he opts to work in the sector with teams, average productivity there rises and, as we shall see, some of the benefits accrue to individuals besides himself. If he decides instead to work in the individualistic sector, the individual captures all of the returns to his talent. Thus, national income would be augmented by a marginal increase in the number of individuals who choose team production, starting from the competitive equilibrium. Since trade encourages further specialization in individualistic production in the country with a more diverse talent pool, it has the potential to reduce national income even as it worsens the distribution of that income. Production subsidies (or tariffs) could reverse these effects, although Pareto improvements are difficult to come by in view of the asymmetries of information that eliminate the possibility of targeted lump-sum compensations.

The remainder of the paper is in five sections. The basic model is developed in Section 2. It has two sectors, one where team production is essential and another in which individuals can work alone. Labor is the sole input to production, but the labor force is heterogeneous. I examine two benchmark equilibria, one that arises when abilities are observable and an equivalent one that exists when ability is private information but team output is verifiable. Section 3 focuses on a small, price-taking country with imperfect labor contracts. I describe a general equilibrium with endogenous occupational choice and explore the links between allocations, incomes, and the exogenous relative price. I also establish the inefficiency of the competitive equilibrium and discuss the policy implications of this. Trade patterns are the subject of Section 4. I consider trade between two large countries that differ only in their distributions of talent. In Section 5 I discuss the relationship of this work to some others in the literature. Section 6 concludes.

2 The Model

The economy has two sectors. In one sector, production is a collective enterprise. A team of workers is needed to perform a set of indivisible tasks, with one worker required for each task. The technology dictates the total number of tasks and thus the size of any production unit. Output generated by a team is $F(q_1, q_2, \ldots, q_n)$, where n is the number of tasks and q_i is the skill of the team member who performs task i. The physical quantity of output may vary with the composition of a team, or the quality of the product may be different for different teams, with $F(\cdot)$ measuring output in quality-adjusted units. In any event, there are no identifiable outputs of the individual contributors, only the joint product of the team. I will refer to this as the "automobile" industry.

In the other sector, individuals can work alone. This may mean that a worker can produce a finished good or service single-handedly, as when a particular investment adviser handles a client's account, or that an individual's contribution to a group effort can be identified separately, as when some person can take credit for the authorship of a particular piece of software. The important assumption is that each worker's output is measurable and verifiable, so that in principle he could operate on his own. I call this the "software" sector.[2]

[2] The designations should not be taken too literally. Although many software firms are small, with one or a few individuals writing specialized code, Microsoft has become a massive company with many team projects. Also, I take liberties in assuming the pervasiveness of synergies in one sector and their complete absence in the other. In reality, some synergies exist between workers in

Both technologies have constant returns to talent. Thus, the potential output of software by an individual of ability q_j is λq_j. Viewed alone, this statement is nothing more than a definition of a unit of talent. But then the operative assumption is that $F(\cdot)$ is homogeneous of degree one when talent is measured in this way. Rosen (1981) and Murphy et al. (1991) have emphasized that "superstars" will be drawn to activities with increasing returns to talent. I do not deny that returns to talent may differ in different activities, or that this consideration has an important impact on the allocation of talent. But there is no *a priori* reason to associate team production with decreasing returns to talent, *when the abilities of all members of the team are increased together.* Accordingly, I make the more neutral assumption that output varies with skill similarly in the two sectors.

I also assume that $F(\cdot)$ is symmetric and set the number of tasks equal to two. The qualitative properties of the model with two members per team are the same as those with larger teams, so there is no need to carry around the extra terms. As for symmetry, it is obvious that skill is more important for some tasks than for others, and that some individuals are especially well suited to perform certain tasks. But the symmetry assumption allows us to focus on issues to do with imperfect contracting without confounding them with considerations of comparative advantage.[3] In this model, a worker of given talents would be equally adept at performing all jobs in a world of perfect information. All of the predictions about occupational choice stem from the assumed informational asymmetries and the restrictions on feasible contracts.

Finally, I take $F(q_1, q_2)$ to be a non-decreasing, twice differentiable, and supermodular function of its two arguments, with an elasticity of substitution between talents that is everywhere less than or equal to one.[4] Supermodularity means that for any four workers, aggregate output is highest when the more able of the two work-

most productive ventures.

[3]See Mussa (1982), Ruffin (1988), and Matsuyama (1992) for trade models in which workers differ in their relative ability to supply labor to different sectors.

[4]This last assumption requires $FF_{12}/F_1F_2 \geq 1$ for all q_1 and q_2.

ers performing task 1 is teamed with the more able of the workers performing task 2, as compared to the alternative possible pairing (see Milgrom and Roberts, 1990). Together with the symmetry of tasks, it implies that it is efficient to pair the two workers of highest ability and the two of lowest ability, for any conceivable foursome working in the automobile industry. With $F(\cdot)$ twice differentiable, the supermodularity assumption is equivalent to $F_{12} \geq 0$. Thus, it captures the idea that team members' talents are complementary in producing value. Further, the bound on the elasticity of substitution ensures that the complementarities are moderately strong. When the elasticity of substitution is never greater than one, both tasks must be completed at a non-zero level of competence for output to be positive. This seems a reasonable restriction to place on what we would call 'team production'.

The labor force comprises a continuum of workers. Each individual is endowed with enough time to perform one productive task, be it one of the tasks needed to produce an automobile or the solo task of writing software. The individuals have no other valuable uses for their time. It takes no time, however, to offer contracts, sell output, or pay wages. Therefore, the same individuals conceivably can own and operate firms.

The distribution of abilities is exogenous in the model. Let $\Phi(q)$ be the fraction of the L workers in the home country with ability less than or equal to q. When there are two countries, $\Phi^*(q)$ will be used to represent the cumulative distribution function in the foreign country, and L^* the labor force there. Often, I will take the distributions to be continuous and differentiable. Then $\phi(q)$ and $\phi^*(q)$ will denote the derivatives, that is the p.d.f.'s for talent in each country.

I assume that all workers in both countries have identical and homothetic preferences. These are represented by the utility function $U(c_a, c_s)$, where c_i is consumption of good i, for $i = a$ (automobiles) and s (software). I also assume for expositional simplicity that individuals are risk neutral, so that $U(\cdot)$ is homogeneous of degree one. However, nothing of importance hinges on this assumption.

Finally, I treat the operation of the labor market as a two-stage occupational-choice-cum-auction game. In the first stage, each individual selects the sector to which he will devote his time. In the second stage, individuals who have chosen to work in the automobile industry are allocated to employers by a bidding mechanism. Potential employers bid for workers by offering employment contracts. The terms of these contract differ depending on the informational setting. For example, if ability is observable, then a contract may specify a wage for a particular type (ability) of worker. If ability is not observable, compensation can be made contingent only on team productivity, or perhaps on nothing at all. After the bids have been submitted, the highest bids in each contract category are designated as winners, and employees are allocated randomly from among those who are interested in and qualified for the jobs. The details of this are described separately for each of the informational settings considered below.

Once occupational choices and team pairings have been determined, production takes place and the (competitive) product market clears.

2.1 Benchmark I: Complete Information

Let us begin with the case in which talents are observable. In this setting, the technology admits a full Walrasian equilibrium. By a "full Walrasian equilibrium", I mean the outcome that would obtain if a disinterested auctioneer were to call out labor contracts and individuals were to submit supplies and demands taking the terms of the contracts as given. This procedure differs from the game of occupational choice specified above. After describing the Walrasian equilibrium, I will argue that the same allocation can be sustained as an equilibrium in the occupational-choice game.

As usual, a competitive Walrasian equilibrium maximizes the value of aggregate output given prices. This implies productive efficiency. Efficiency in turn requires positive assortative matching in the automobile industry; that is, each team producing cars comprises two workers of exactly equal ability.[5]

[5]This follows directly from the definition of supermodularity and the symmetry assumption; see

Let $f \equiv F(1,1)/2$. Then, since $F(\cdot)$ has constant returns to talent, $2fq$ is the output of a pair of workers of talent q. The same two workers could produce $2\lambda q$ units of software. Thus, with efficient matching, the productivity of every worker is proportional to his talent, regardless of his sector of employment. This means that each country has a linear production possibility frontier with slope f/λ. The Walrasian equilibrium is a Ricardian trade equilibrium among two countries with identical technologies; consequently, there is no trade.[6]

In the Walrasian equilibrium, the allocation of talent is indeterminate. The market clearing contracts pay wages that are proportional to ability. A worker of ability q is offered λpq for a job in the software industry and fq for one in the automobile industry, where p is the relative price of software in terms of autos. As in any Ricardian setting, the equilibrium price must be $p = f/\lambda$ for positive output of both goods. Then each worker is indifferent regarding his sector of employment. Only the aggregate allocation of talent to each sector is determined, and in such a way that the product market clears in each country.

Now consider the game of occupational choice. This game has a subgame perfect equilibrium that achieves the same allocation as the Walrasian equilibrium. In this equilibrium, the same set of workers selects employment in the automobile industry as in the Walrasian equilibrium. Each individual of ability q offers fq for a partner of the same ability as himself. Half of these bids are designated as winners, and the remaining half of the individuals are assigned as employees to the winning bidders for their type.[7] With these strategies, no individual who selects the automobile sector can improve his lot by altering his bidding strategy, nor can any individual benefit from choosing a different sector in the initial stage.

Kremer (1993) or Grossman and Maggi (1998).

[6] Actually, inessential trade might occur, because each country's offer curve is perfectly elastic for a range of trades at the common autarky price. However, a tiny international transport cost would eliminate all such trade.

[7] Alternatively, bids may be submitted by third parties, who offer fq for a pair of workers of ability q. Since there are no profits or informational rents to be had, the identity of the employers is indeterminate and immaterial.

2.2 Benchmark II: Performance-Based Contracts

I examine a second benchmark situation in which neither an individual's ability nor his contribution to team productivity are observable to his employer or to a court of law. However, firms and workers can readily monitor and verify the (quality-adjusted) output of every team. With output verifiable, employers can write performance-based contracts. Can such contracts generate the efficient allocation of talent in a competitive equilibrium?

It might seem that, without personal performance incentives, the most talented workers could not get their due in the automobile sector. In fact, an efficient allocation can be achieved provided that the employment contracts are tendered by third parties. These third parties are individuals who own and operate firms but take no part themselves in the firm's production. Instead, they act as residual claimants, paying each employee a wage based on his team's output and keeping any remaining profits for themselves. The bids are such that workers reveal their types, and each is paired with another of exactly the same ability.

I describe an equilibrium in which both the allocation of talent to sectors and the pairings within the automobile industry match those in the full-information equilibrium. In this equilibrium, the initial choices mimic those in the full Walrasian equilibrium. Then an arbitrary set of individuals bids for pairs of workers with contracts that promise each team member a wage of fq if joint output by the pair is $2fq$, and zero if joint output is anything else. There are more offers than there are available employees for each talent level q, and a continuum of contract types covering all of the possible values of q. Those seeking employment apply only for the jobs targeted to their ability, and each worker of talent q indeed earns fq once output is realized. Notice that no worker could benefit from applying for a different set of jobs, no bidder could benefit from altering contract terms, and no individual could benefit from making a different occupational choice. In the equilibrium, all firm owners earn zero profits.

Once again, I have described an equilibrium without any trade.[8] In this equilibrium, incentive contracts induce individuals to reveal their talent, which in turn makes efficient matching possible. When team pairings are efficient, the output of each good is proportional to aggregate talent in the industry. Then countries with similar technologies have similar production possibilities, regardless of how talent is distributed in their labor forces.

3 Imperfect Labor Contracts

Now I assume that an individual's ability is not observable and that a team's output cannot be verified. For example, it may be difficult for a court to judge the quality of an automobile, or to ascertain which cars were produced by a given team. Then contracts linking pay to productivity are impossible to enforce. Potential employers have no choice but to offer contracts with fixed wages. These contracts are imperfect here, because they cannot be used to generate the efficient matches in the automobile industry. In this section, I study the general equilibrium for a small country that takes the world price of software as given.

In the equilibrium with imperfect contracts, individuals sort themselves by talent into different industries and different roles. The basis for the sorting is illustrated in Figure 1, which depicts income as a function of talent for three different options open to any individual.

First, an individual might choose to enter the automobile industry with the intention of becoming an employee there. Then he would receive a fixed wage w,

[8] There does exist an equilibrium with trade, although it requires especially pessimistic expectations in one (and only one) country. Suppose that, at the moment of occupational choice, every individual in one country anticipates that all others will opt to work alone. Then each individual would be justified in choosing the software industry himself, inasmuch as he would be unable to find a partner were he to select the other. This country would specialize in software production, while the other would produce all of the world's cars. Since there is no reason why expectations should be so pessimistic in one country but not in the other, this hardly seems a compelling explanation for trade.

independent of his talent, as represented by the horizontal line WW in the figure.

Second, the individual might enter the automobile industry, but with the intention of hiring another to form a team. Such employers face uncertain prospects, because they commit to pay a fixed wage while their proceeds depend on the identity of the partner with whom they are matched. But for any pool of potential employees, a prospective employer can compute the expected output of a team on which he might participate. Expected income is just the difference between expected output and the promised wage. The curve MM depicts the relationship between expected income and ability when the expected wage is w. Notice that expected income rises with ability, but does so at a diminishing rate. This is because each employer draws from a pool with a given distribution of talent, so expected output does not increase in proportion to the employer's own ability.

Finally, an individual may choose to enter the software sector. In this case, productivity and income do rise in proportion to talent. This relationship between income and ability is depicted by EE in the figure.

In the equilibrium, the most talented people opt to enter the software sector. The returns to talent are highest there, because income does not depend on the contribution of a partner drawn from a given pool of talent. Among the remaining population, the more able individuals prefer to be employers while the less able serve as employees. The employers—whom I term "managers"—choose to become residual claimants, because they know that they have sufficient ability to produce an expected output in excess of $2w$. The less able — who become "workers"— know the opposite to be true. They are happy to accept w rather than hire another at that wage. Finally, the equilibrium leaves no opportunities for third parties to hire workers in the automobile industry, because expected profits are negative when a pair of hires is drawn randomly from the employee pool and each worker must be paid the equilibrium wage w.[9]

[9]The allocation I have just described requires an equilibrium wage such that the WW curve passes above the left-most intersection of the MM and EE curves; otherwise, a group of individuals with talent just above that of the workers would earn greater expected income in the software

With this intuition in mind, I turn to a more formal statement of the conditions that must be satisfied in an equilibrium. Let $[q_{\min}, q_{\max}]$ be the support of the talent distribution Φ, and let q_w and q_m be the abilities of the most talented worker and the most talented manager, respectively. Then, in equilibrium, individuals with abilities between $q_{\min}$ and q_m enter the automobile industry, while those with abilities between q_m and $q_{\max}$ elect to produce software. Assume, provisionally, that output in both sectors is positive; i.e., that $q_{\min} < q_w < q_m < q_{\max}$. Then the most talented worker, with talent q_w, must be indifferent between the sure wage w and the expected income he would earn by hiring a random partner from the pool of potential workers and paying that partner w . This implies

$$w = \frac{1}{\Phi(q_w)} \int_{q_{\min}}^{q_w} F(q_w, q) d\Phi(q) - w. \tag{1}$$

The most talented manager must be indifferent between his expected income in the automobile industry and what he could earn by entering the software sector. This indifference can be expressed as

$$\lambda p q_m = \frac{1}{\Phi(q_w)} \int_{q_{\min}}^{q_w} F(q_m, q) d\Phi(q) - w. \tag{2}$$

Finally, the number of employers in the automobile sector must match the number of employees, so that each manager is with a single worker. This requires

$$\Phi(q_w) = \Phi(q_m) - \Phi(q_w). \tag{3}$$

Suppose that values of q_m, q_w and w can be found that satisfy (1), (2) and (3) such that $q_{\max} > q_m > q_w > q_{\min}$ and $w \geq 0$. Suppose further that, at these values, $\tilde{F}(z) - w > \lambda p z$ for all $z \in (q_w, q_m)$, where $\tilde{F}(z) \equiv \frac{1}{\Phi(q_w)} \int_{q_{\min}}^{q_w} F(z, q) d\Phi(q)$ is the expected output of an automobile firm managed by a worker of ability z when the worker pool consists of all workers with abilities between $q_{\min}$ and q_w. I claim that

industry than they would as managers of automobile plants. I will show that the software industry indeed attracts *only* the most talented workers in the economy when the complementarities in team production are sufficiently strong. For this outcome, it is sufficient that $F(\cdot)$ has an elasticity of subsitution everywhere less than or equal to one, which I have assumed to be the case.

these values then constitute an equilibrium in the occupational-choice game. In other words, there exist occupational choices and bidding strategies for all individuals such that all winning bids for automobile workers pay exactly w and an individual enters the automobile industry if and only if his talent lies between $q_{\min}$ and q_m.

To establish this claim, I first take the occupational choices as given. Suppose there is a measure L_a of individuals who have entered the automobile industry. Each such entrant believes that the others are among the L_a least talented individuals in the economy. All those with talent above the median in this (suspected) group bid the wage that would make the median individual in the group indifferent between being an employee and an employer. Meanwhile, each individual with talent below the group median bids the wage that leaves him personally indifferent between working as an employee or hiring a random partner from the employee pool (those with below median talent) at that wage. Third parties do not bid at all. With these strategies, when the automobile industry does indeed comprise the least talented L_a individuals in the economy, none has any incentive to deviate. The more talented half become the winning bidders. Each of them pays just what is needed to hire an employee, but nothing more. A higher bid would only serve to raise the wage bill, while a lower bid would drop the individual from the winning set. Since the designated winning bidders have talent at least as great as the suspected median in the group, each strictly prefers to hire another at the specified wage than to be hired himself. As for the losing bidders, they cannot benefit by bidding more (since they have bid their reservation wages), nor do they have any reason to bid less. And no third party would wish to submit a winning bid, because the expected productivity of two random hires from among a worker pool comprising the least talented $L_a/2$ workers is less than twice the wage that would leave the median in the group indifferent between acting as worker or as a manager.

Turning to the first-stage decisions, each individual with talent between q_m and $q_{\max}$ earns more in the software industry than he could expect to earn by entering the automobile sector and submitting a winning bid there. Meanwhile, all those with talents between $q_{\min}$ and q_m have expected incomes greater than $\lambda p q_i$, which is what they could earn by producing software.

If there is no solution to (1), (2) and (3) with $q_{\max} > q_m > q_w > q_{\min}$, $w \geq 0$, and $\tilde{F}(z) - w > \lambda p z$ for all $z \in (q_w, q_m)$, then either the equilibrium has a group of individuals in the software industry with ability less than that of some managers, the equilibrium is one with complete specialization, or no equilibrium exists. I show, in Appendix A, that an equilibrium always exists, and that no equilibrium has any individuals in the software sector with ability less than that of the most talented manager. This leaves complete specialization as the only remaining possibility.

Specialization in the automobile industry requires that the most talented individual earns more by hiring a random partner from among those in the bottom half of the talent distribution than he could by working alone in the software industry. With specialization in auto production, the equilibrium wage would be $w = \int_{q_{\min}}^{q_{med}} F(q_{med}, q) d\Phi(q)$, in view of the condition of indifference for the marginal worker, where q_{med} is the talent of the median individual. Then the most talented individual indeed prefers to enter the automobile industry if and only if

$$2 \int_{q_{\min}}^{q_{med}} F(q_{\max}, q) d\Phi(q) - \int_{q_{\min}}^{q_{med}} F(q_{med}, q) d\Phi(q) \geq \lambda p q_{\max} \; .$$

Specialization in the software industry arises when the least talented worker can earn more by producing software than what he expects to earn by entering the automobile sector. An equilibrium like this always exists; if each individual believes that no others will enter the automobile sector, than each sees no prospect for finding a partner there. Such beliefs are most plausible, however, when even the least-talented pair of workers could earn more in the software sector than they could by producing cars and sharing the output. The condition for this is $\lambda p q_{\min} > f q_{\min}$, or $\lambda p > f$.

In Figure 2, I show aggregate output of software, x_s, as a function of the relative price p. Naturally, output of software varies inversely with the size of the automobile sector, as measured by q_m. More specifically, $x_s = \lambda L \int_{q_m}^{q_{\max}} q d\Phi(q)$. For low values of

p such that

$$p < p_a \equiv \frac{2\int_{q_{\min}}^{q_{med}} F(q_{\max}, q)d\Phi(q) - \int_{q_{\min}}^{q_{med}} F(q_{med}, q)d\Phi(q)}{\lambda q_{\max}} \tag{4}$$

there exists only an equilibrium with $q_m = q_{\max}$ and $x_s = 0$. For high values of p such that $p > p_s \equiv f/\lambda$, the only equilibrium has $q_m = q_{\min}$ and $x_a = 0$. Then $x_s = \lambda L\bar{q}$, where $\bar{q} \equiv \int_{q_{\min}}^{q_{\max}} qd\Phi(q)$ is the average talent level in the economy. It can be shown that $p_s > p_a$, as depicted in the figure.[10] Finally, for $p \in [p_a, p_s]$ there must be at least one solution to (1), (2) and (3) with $\tilde{F}(z) - w > \lambda pz$, and there might be more. For each such solution, x_s can be calculated from q_m. Panel (a) depicts the case of a unique solution to the system for each value of p, while panel (b) shows the case where multiple solutions exist for some values of p. In Appendix B I show that panel (a) must apply when talent is uniformly distributed.

3.1 Exogenous changes in relative price

Now consider the effects of an increase in p, as for example when the terms of trade improve in a country with a comparative advantage in software. Figure 3 will prove useful for this purpose. In this figure, the curve AA depicts combinations of q_m and w that satisfy (1) and (3) for a given price p between p_a and p_s. Along this curve, when all individuals with ability less than or equal to q_m work in the automobile sector and exactly half of these are managers, the individual with talent q_w is just indifferent between being a worker and being a manager. The curve slopes upward, because the greater is the wage, the more tempting it is to be a worker, and only an individual of greater ability choosing from a more talented employment pool would be indifferent between the two roles. The curve SS in turn depicts the combinations of w and q_m

[10] Note that $p_s > p_a$ if and only if

$$fq_{\max} > 2\int_{q_{\min}}^{q_{med}} \left[F(q_{\max}, q) - \frac{1}{2}\ F(q_{med}, q)\right] d\Phi(q).$$

But $fq_{\max} = 2\int_{q_{\min}}^{q_{med}} \frac{1}{2}F(q_{\max}, q_{\max})d\Phi(q)$, so $p_s > p_a$ if and only if $\int_{q_{\min}}^{q_{med}} I(q)d\Phi(q) > 0$, where $I(q) \equiv \frac{1}{2}F(q_{\max}, q_{\max}) + \frac{1}{2}F(q_{med}, q) - F(q_{\max}, q)$. Now, $I(q_{med}) > 0$ by the supermodularity of F, and $I'(q) = \frac{1}{2}F_2(q_{med}, q) - F_2(q_{\max}, q) < 0$. Therefore $\int_{q_{\min}}^{q_{med}} I(q)d\Phi(q) > 0$, which implies $p_s > p_a$.

that satisfy (2) and (3); i.e., they make the marginal manager indifferent between entering the software and automobile industries after taking into account who would be in the automobile employment pool at the given q_m. This curve can slope in either direction, and can be steeper of flatter than the AA curve when it is upward sloping. To see this, note that expected income rises with ability in both sectors. In the software industry, expected income rises exactly in proportion to talent. But in the automobile industry, expected income may rise more than or less than in proportion to the talent of the best manager, *once the associated change in the employment pool is taken into account*. If the best manager's expected income from running an automobile firm rises less than in proportion to talent after accounting for the change in q_w, then a new marginal manager with greater talent can be indifferent between industries only if the wage is lower. This gives a downward sloping SS curve, as depicted in panel (a). If, on the other hand, income for the most talented manager in the automobile sector rises more than in proportion to q_m after q_w adjusts, then the SS curve slopes upward. The curve must lie above the AA curve at $q_m = q_{\min}$ and it must lie below it at $q_m = q_{\max}$, but, in principle, it can cross the AA curve several times.[11] Multiple crossings, such as are depicted in panel (b), correspond to the multiplicity of possible outputs of software for a given price, as shown in panel (b) of Figure 2.

An increase in the price of software makes employment in the software sector more attractive. An individual with some given talent who was indifferent between managing an automobile firm and working alone in the software sector before the price change will only be indifferent now if the cost of hiring an employee in the automobile sector is lower. Thus, the SS curve shifts downward, as indicated by the dotted curves in the two panels of Figure 3. Meanwhile, the price hike leaves the AA curve unaffected.

When the equilibrium is unique, as for example when talent is distributed uniformly, an increase in the price of software causes q_m to fall. This is clear in panel

[11]The fact that $p > p_a$ ensures SS above AA at $q_m = q_{\min}$. Similarly, $p < p_s$ ensures AA above SS at $q_m = q_{\max}$.

(a), which depicts a falling SS curve, and it also applies to the case of a rising SS curve that cuts the AA curve once from below.[12] Intuitively, an increase in p draws individuals into the software sector by improving the prospects there for those who might otherwise operate automobile firms. As the top managers leave the automobile industry, some who were workers must now operate their own firms. That is, q_w falls, and with it the average talent of those remaining in the employment pool. The wage rate is equal to one-half the expected output of the least-talented manager and his random partner (see (1)). Since both q_w and the average ability of the partner fall, the wage falls as well.

Figure 4 depicts two possible shifts in the talent-income profile. In both cases, the rich get richer, while the poor get poorer! The most talented individuals — those who toiled in the software industry in the original equilibrium — surely benefit from any increase in the relative price of software. Their nominal incomes rise in proportion to p, which means that their real incomes rise no matter what they consume. And the least talented individuals — those who are employees in automobile firms in the final equilibrium — surely lose. For these individuals, nominal incomes fall even as consumer prices rise. In panel (a), the rise in p hurts all those who remain in the automobile sector after the change in relative price. Here the benefits of the terms of trade improvement go only to the society's elite. Panel (b) shows the possibility that a group of relatively low-ability managers in the automobile industry might benefit from a rise in p. For these individuals, the cost savings from the drop in wages might outweigh the (expected) loss of productivity from the degradation of the employment pool.[13]

Figure 3b shows the effects of an increase in the price of software when there are multiple equilibria. Starting from any equilibrium at which the SS curve cuts the AA

[12] If the curves cross only once, the SS curve must be flatter than the AA curve at the point of intersection, by the argument of the previous footnote.

[13] Note that the decline in the wage benefits all managers of automobile firms equally, whereas the degredation of the talent pool hurts most the managers with the greatest talent. Therefore, it is the least talented of the original managers, if any, who stand to benefit from the net effect of the two.

curve from below, the comparative statics are qualitatively like the ones I have just described; employment in the software sector expands, output of software rises, and the wage rate falls. All of these responses are reversed when the AA curve is steeper than the SS curve at the initial equilibrium. It is worth noting, however, that such equilibria are unstable for an ad hoc adjustment process under which the wage rises when the median individual in the automobile industry earns more as a worker than as a manager (given q_m) and the automobile sector contracts when the most talented manager could earn more in that sector than he could by hiring an employee, given w.

3.2 (In)efficiency of the free-trade equilibrium

I will now argue that a subsidy to automobile producers would expand the size of the economic pie even as it redistributes income from those who earn the most to those who earn the least.[14] In other words, the size of the software sector in the free-trade equilibrium is larger than that which maximizes the value of national output at international prices.

The argument is straightforward. At the free-trade equilibrium, the least talented software writer produces output worth $\lambda p q_m$. If he were to choose instead to enter the automobile industry, his marginal contribution to national income would be $\pi(q_m) + \Omega/2\Phi(q_w)$, where $\pi(q_m)$ is the expected income of a manager of talent q_m, and[15]

$$\Omega = \int_{q_w}^{q_m} \left[F(z, q_w) - \frac{1}{\Phi(q_w)} \int_{q_{\min}}^{q_w} F(z, q)\phi(q)dq \right] \phi(z)dz.$$

But $\pi(q_m) = \lambda p q_m$, since the marginal manager must be indifferent between the two industries when occupational choices are made to maximize personal income. National income would be augmented by shifting the marginal software writer to the

[14] In making this statement, I neglect the possibility that the initial equilibrium is one at which the AA curve cuts the SS curve from below.

[15] Note that $x_a = \frac{L}{\Phi(q_w)} \int_{q_w}^{q_m} \left[\int_{q_{\min}}^{q_w} F(z, q)d\Phi(q) \right] d\Phi(z)$. The marginal contribution to national income is $(dx_a/dq_m)/\phi(q_m)$, which can be calculated directly using $dq_w/dq_m = \phi(q_m)/2\phi(q_w)$.

automobile industry if and only if $\Omega > 0$. In fact, Ω must be positive, because the term in brackets is the difference in expected output when a manager of talent z teams with the most talented worker compared to when he teams with a randomly selected worker.

I conclude that, when labor contracts are imperfect, private and social incentives diverge. When a talented individual enters into team production, he generates a positive externality. This is because his presence in the industry improves the talent pool there, which raises the average productivity in firms other than his own. In contrast, a talented individual appropriates all of the social benefits when he elects to work alone. A government can subsidize the team sector to encourage entry there. Not only would the poorest members of society benefit, but the increase in income would exceed the cost of the subsidy.

4 Talent Distribution and the Pattern of Trade

In the last section, I established some properties of a trade equilibrium in a small country with imperfect labor contracts. This section examines the pattern of trade between two large countries. More specifically, I link the trade pattern to differences in the distribution of talent. Once that is done, I will be able to discuss how trade affects income distribution differently in relatively homogeneous versus relatively heterogeneous societies.

I consider the special case in which each country has a uniform distribution of talent and the mean skill levels in the two countries are the same. In the foreign country, the range of talents is from $\bar{q} - e^*$ to $\bar{q} + e^*$. In the home country, talents run from $\bar{q} - e$ to $\bar{q} + e$, with $e > e^*$. Thus, the home country has a more diverse labor force than the foreign country. We will see how this leads the home country to export software.

In a free-trade equilibrium, the countries face the same relative world price. We can ascertain the trade pattern by examining how the relative supplies of the two goods respond to a change in e at a given price p.

To this end, consider Figure 5. The solid lines in the figure show the determination of the marginal manager and the wage rate for an initial value of e. The two curves have the same interpretation as in Figure 3; AA depicts combinations of q_m and w that leave the individual with talent q_w indifferent between being a worker and a manager when half of the individuals in the automobile sector are managers, while SS shows combinations of q_m and w that make the marginal manager indifferent between entering the two sectors. Recall that the AA curve slopes upward no matter what the distribution of talent, while the SS curve slopes downward when talent is uniformly distributed.

A spread in the distribution of talent shifts both curves downward. The new locations are indicated by the dotted lines. The shift in the AA curve can be understood as follows. For a given value of q_m, an increase in e reduces q_w. If q_m were unchanged, the new least-talented manager would be less productive than before, and, moreover, he would draw from a less talented pool of workers. For this individual to be attracted to managing, the wage would need to be lower than before. As for the SS curve, the reasoning is similar. For an individual with talent q_m, expected revenues in the automobile sector decline with an increase in e, because the average ability of the potential partners is lower. The individual who was initially the marginal manager can be indifferent between the two industries only if the wage rate also is lower.

The algebra reveals that the SS curve shifts down by more.[16] Thus, an increase in the spread of talent at a given price causes some individuals who would have worked in the team sector to opt instead for individualistic production. The downward shift in q_m occurs because the fall in the average ability of a prospective hire outweighs the decline in the wage. This reflects the assumed complementarities in team production. Since the least-talented manager is relatively close in ability to his expected partner, the downward pressure on the wage caused by the dilution of the worker pool is modest compared to the loss of productivity for the most-talented manager, for whom the reduction in a partner's talent is especially damaging.

[16] All of the algebra is relegated to the appendix.

It can be shown, in fact, that $dq_m/de < -1$; i.e., the range of individuals who choose the automobile sector contracts, even as the number of persons with any given talent level falls. Thus, there are fewer individuals in the automobile sector after the mean-preserving spread, and the average ability of both workers and managers is diminished. This implies that a spread in talent reduces output of automobiles at a given price.

In the software sector, the number of writers expands, but average ability falls. Output would remain the same if dq_m/de were equal to minus one. Then the number of software writers would be unchanged, and the average ability would be the same as well. Since $dq_m/de < -1$, the sector is even larger than the size that would keep output constant. Although the extra software writers are less productive than the others in the industry, they still produce positive output. Therefore, a spread in the talent distribution increases aggregate software production.

To summarize, the greater is diversity, the greater is the disincentive for a talented individual to enter into team production when labor contracts are imperfect. Such an individual would draw from an employee pool of lesser average ability, and although the wage of a worker would be lower as well, the loss in expected productivity would be larger than the cost savings. We find that the country with a more diverse population produces relatively fewer automobiles, and of course relatively more software. With identical and homothetic preferences, this country imports automobiles and exports software in a free-trade equilibrium.

What are the effects of the trade induced by imperfect labor contracting? Compared to autarky, the relative price of software is higher in the country that exports software. We have seen that a rise in p raises the income of the most talented (and richest) individuals, while reducing the wage for those with the least ability. Thus, trade contributes to a further polarization of society in a country that has a relatively diverse labor force. Just the opposite is true in the relatively homogeneous society; the range of incomes was relatively narrow to begin with, and trade narrows this range even further. Finally, note that trade exacerbates the informational externality in the country with a greater spread of talents, whereas it alleviates this externality in its trade partner.

5 Related Literature

Murphy et al. (1991) contains an excellent discussion of the factors that guide the allocation of talent. In their modeling of occupational choice, these authors follow Rosen (1981) in emphasizing returns to scale. They point out that the ablest people tend to choose sectors with large potential markets and weak diminishing returns to scale. This allows a "superstar" to spread his ability advantage over the largest possible scale of operations. Murphy et al. present a model, based on Lucas (1978), in which there is a continuum of individuals with heterogeneous abilities. Each production unit has a single manager and an endogenous workforce, where the productivity of the workforce is proportional to the ability of the manager. In this setting, the most talented individuals become managers, because their extra profits from a given workforce are more than in proportion to their ability advantage, and because the abler managers can operate larger firms and so spread their talent over a larger scale.

Murphy et al. certainly recognized the importance of contract considerations in determining what occupations and sectors would be attractive to talented persons. In fact, they wrote that

> differences in contracts between industries are probably as important or more important than diminishing returns to scale [for determining the allocation of talent]. In industries where it is easy to identify and reward talent, it might be possible to pay the able people the true quasi rents on their ability and so attract them. ... Starting one's own company is obviously the most direct way to capitalize on one's talent without sharing the quasi rents. ... Also, talent will flow into sectors with less joint production, where it is easier to assign credit and reward contributions. (p.513)

In this paper I have developed a general equilibrium model of occupational choice in which imperfect contracting governs the choice of job and sector by individuals of different abilities. My model complements that of Lucas (1978) where potential scale plays the critical role.

The issues to do with adverse selection in my analysis call to mind some of the literature on efficiency wages. In particular, Weiss (1980, 1991) and Malcomson (1981) present models in which firms that cannot observe workers' abilities offer "extra" wages in order to make their jobs appealing to a wider range of talents.[17] The focus of these papers is on the unemployment that could result when all firms attempt to pay above-market wages in order to improve their applicant pool. Here, I have intentionally adopted an institutional setting in which efficiency wages are impossible. First, I have assumed ex post immobility across sectors, which means that the population of workers in the automobile industry is fixed at the time that contracts are tendered. Second, I have modeled the labor market as a multi-winner auction, which precludes talented managers from offering high wages and thereby attracting applications from individuals who would otherwise serve as managers themselves. My intent here was to focus on the allocation of talent in a simple model with imperfect contracts, not to study the determinants of unemployment. Since perfect matching could never result even if firms were to offer efficiency wages, it seemed best to abstract from the complications that such offers introduce.[18]

My analysis of the trade equilibrium bears a family resemblance to that in Clemenz (1995). Clemenz studies a two-sector model in which firms can observe a worker's productivity in one sector but not the other. In the sector with unobservable productivity, firms pay efficiency wages. Those who cannot secure jobs in this sector at the

[17] Theirs are not general equilibrium models of occupational choice in the sense that mine is, inasmuch as the indentities of the residual are not determined endogenously in the model.

[18] In a setting in which talented managers can offer high wages to attract a more talented pool of applicants, there would still be no way to prevent the less talented from applying for the high-wage jobs. Thus, perfect matching of individuals of like ability in the automobile industry could not be an equilibrium outcome.

above-market clearing wage find employment ex post in the other sector. Clemenz investigates the free-trade equilibrium that results when there are two types of workers in each country. He finds that the country with the greater proportion of high-ability individuals has a comparative advantage in the sector where information is imperfect. He also concludes, like me, that the equilibrium allocation of talent is inefficient, and that trade can bring harm to one of the countries by causing its sector with unobservable productivity to contract.

This paper also relates to previous research on the matching of workers in firms. In particular, Kremer (1993) and Legros and Newman (1997) have studied the sorting generated by a competitive labor market when the production process requires that several individuals interact and the tasks performed by each are complementary in producing output. Kremer and Maskin (1996) document the growing segregation of American workers by skill and the absolute decline in wages of low-skill workers. They use a model with complementarities between asymmetric tasks to explain these observations, which they ascribe ultimately to an increase in both the mean and the dispersion of talents in the U.S. labor force. My results suggest that growing trade with countries that have more homogeneous populations than the United States can account for many of the same observations.

Like in this paper, Grossman and Maggi (1998) draw a link between the diversity of talent in a country's labor force and the pattern of its international trade. But they emphasize technological differences in the interaction between workers in different sectors. In some industries, different productive activities may be complementary, as suggested by Kremer (1993). In other industries, different activities may be substitutable in creating output; for example, value may be very high when one or a small number of tasks is performed especially well. If this is true, then a country with a more diverse labor force has a comparative advantage in the sector in which there is substitutability between tasks. Since substitutability often makes working alone optimal, the technological explanation of the trade pattern complements the contracting story offered here.

6 Conclusions

Compensation contracts take different forms in the different sectors of an economy. In some sectors, individuals can reap enormous gains if their contributions prove highly valuable. In other sectors, a narrower range of rewards is possible. Often these latter are the sectors where it is difficult to attribute profits to individuals. Differences in contracts play an important and neglected role in the allocation of resources.

In this paper, I have developed a simple general equilibrium model with imperfect labor contracts. In one sector, teams produce output the value of which cannot be verified in court. In the other sector, individuals contributions are readily observable, or individuals can work alone. The most talented individuals prefer the second type of industry, because they can capture greater returns to their ability there. This leaves those with moderate or lesser talents to enter into team production.

When countries differ in the compositions of their labor forces, the pressures for this type of segregation by skill vary. The ablest individuals have the greatest incentive to separate themselves when the difference between their own skill and that of their (outwardly similar) compatriots is substantial. While these incentives are present even in a relatively homogeneous society, they are less severe there. It follows that diversity breeds comparative advantage in the face of private information about ability and imperfect labor contracting. A country with a relatively heterogeneous labor force will export goods produced by individual (or attributable) efforts and import those produced by teams. The growing U.S. comparative advantage in financial services and software and the continuing decline of its Rust Belt can perhaps be understood in these terms.

The polarization that accompanies growing trade with more homogeneous societies has a social cost. As the most able people opt for individualistic activities, the talent pool available to teams is diminished. This lowers average productivity in the team sector and drives down wages there. Thus, trade benefits the most talented individuals in the diverse country at the expense of those who are least well off.

My analysis has employed a number of simplifying assumptions. In reality, there are varying degrees of observability of output and profits, and a variety of contract provisions that tie pay to observable performance measures. For example, executives in publicly traded firms often receive stock options as part of their compensation packages. The value of these options varies automatically with the market's assessment of the performance of the firm, though the options do not reflect exactly the executive's personal productivity. An interesting extension of this paper would expand the richness of the contract space to allow finer predictions about the intersectoral allocation of talent.

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Figure 1

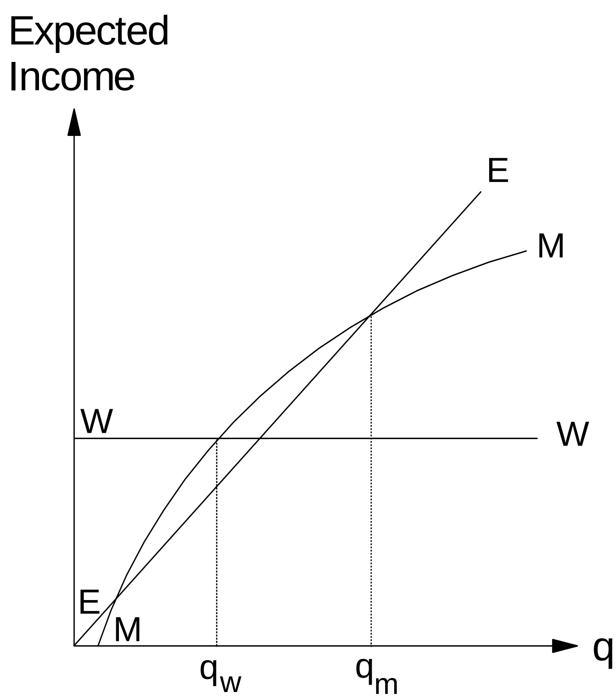

Figure 1:

Figure 2

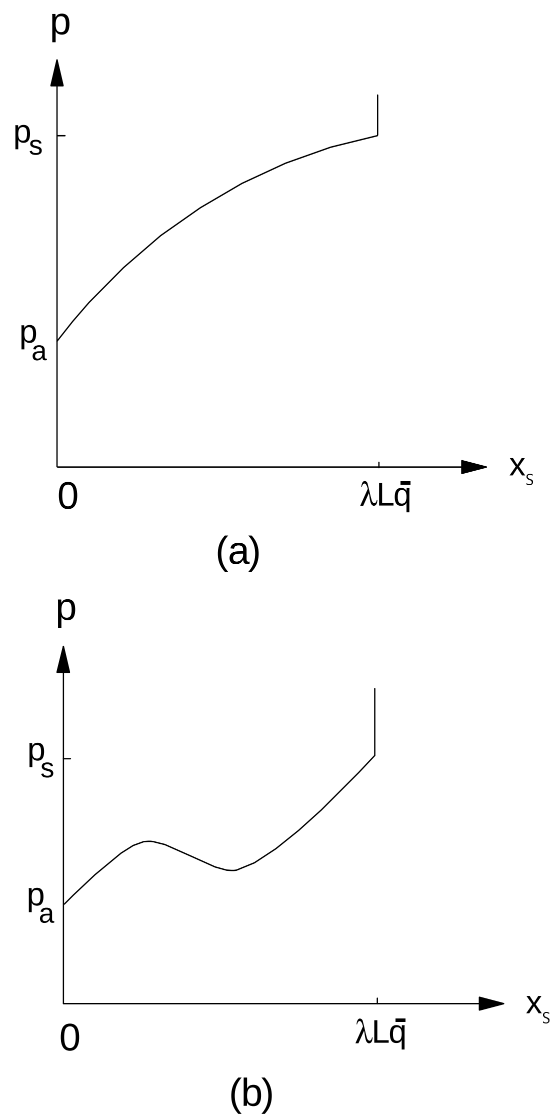

Figure 2:

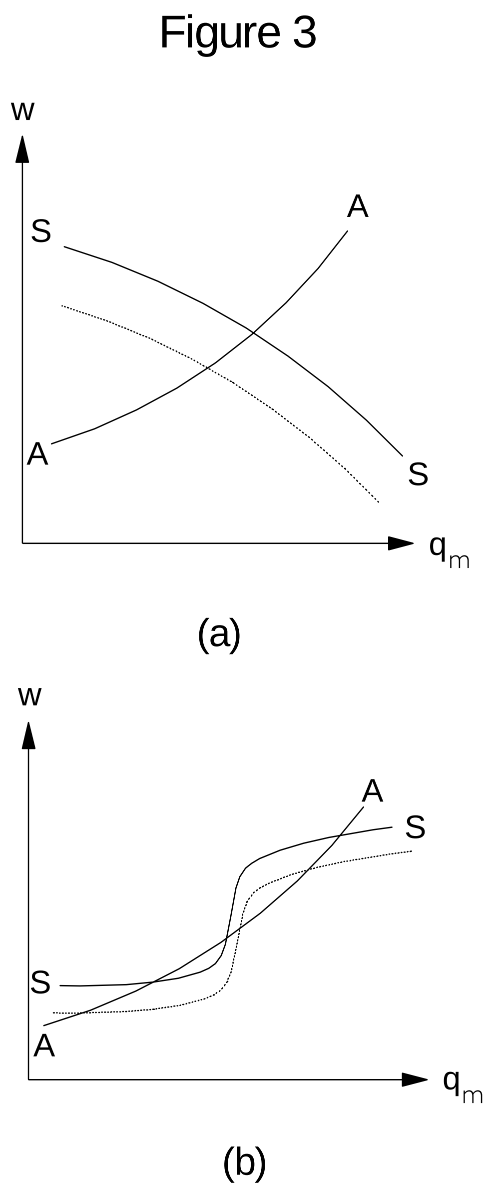

Figure 3:

Figure 4

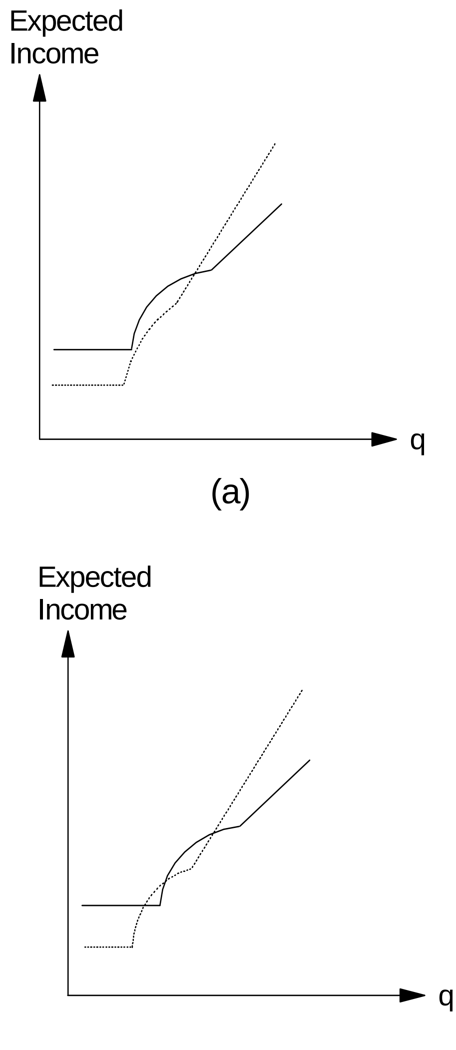

Figure 4:

Figure 5

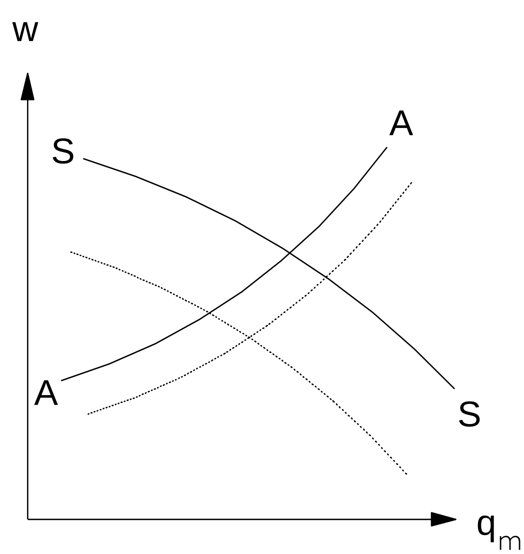

Figure 5:

Figure 6

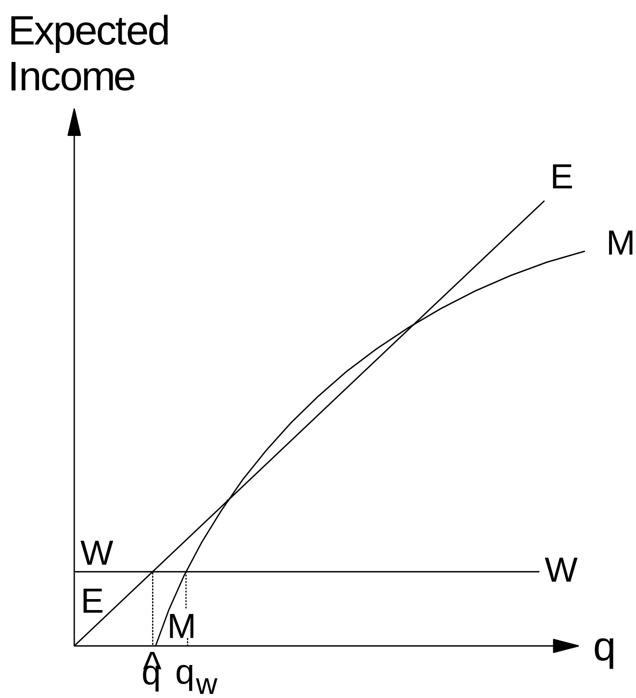

Figure 6:

Figure 7

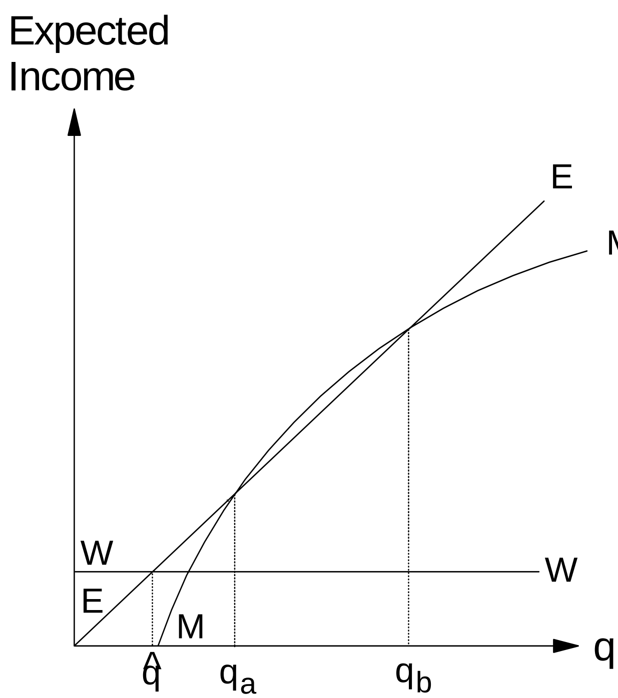

Figure 7:

Appendix A

In this appendix I prove that, for $p \in (p_a, p_s)$, there exists an equilibrium with incomplete specialization in which every individual in the software sector has greater ability than the most able manager in the automobile industry. I also prove that there does not exist any equilibrium in which some individuals in the software industy have less ability than that of the most able manager.

For $p > p_a$, the SS curve of Figure 3 lies above the AA curve at $q_m = q_{\min}$. For $p < p_s$, the AA curve lies above the SS curve at $q_m = q_{\max}$. Both curves are continuous. Therefore, when $p \in (p_a, p_s)$, the curves must intersect at least once for some q_m between $q_{\min}$ and $q_{\max}$. At this intersection, equations (1)-(3) are satisfied and $w > 0$. To establish the existence of an equilibrium of the sort described in the text, it remains to verify only that $\tilde{F}(z) - w > \lambda p z$ for all $z \in (q_w, q_m)$, when w, q_w, and q_m take on the values associated with the point of intersection.

Suppose not. Then, for the values of w, q_w, and q_m that satisfy (1)-(3), the configuration of the WW, MM, and EE curves must be as depicted in Figure 6. Define $\hat{q}$ such that $\lambda p \hat{q} = w$. Notice that $\hat{q} < q_w$, which, with (1), implies $\tilde{F}(q_w)/2q_w < \lambda p$. Notice too that the slope of the MM curve at q_w exceeds its slope at the (first) intersection with EE, which in turn exceeds the slope of EE. Thus,

$$\tilde{F}'(q_w) > \lambda p.$$

But

$$\tilde{F}'(q_w) = \frac{1}{\Phi(q_w)} \int_{q_{\min}}^{q_w} F_1(q_w, q) d\Phi(q).$$

Since $F(\cdot)$ is symmetric and has an elasticity of substitution less than one, $q_w F_1(q_w, q)/F(q_w, q) \leq 1/2$ for all $q \leq q_w$. Therefore

$$\frac{1}{\Phi(q_w)} \int_{q_{\min}}^{q_w} F_1(q_w, q) d\Phi(q) < \frac{1}{\Phi(q_w)} \int_{q_{\min}}^{q_w} \frac{F(q_w, q)}{2q_w} d\Phi(q) d\Phi(q) = \frac{\tilde{F}(q_w)}{2q_w},$$

which is a contradition. It follows that, for $p \in (p_a, p_s)$, there exists a solution to (1)-(3) with $w > 0$ and $\tilde{F}(z) - w > \lambda p z$ for all $z \in (q_w, q_m)$.

Now suppose that there exists an equilibrium with incomplete specialization in which some individuals in the software industry have less ability than that of the most able manager. In such an equilibrium, the allocation of labor must be as shown in Figure 7. The least talented individuals with $q < \hat{q} = w/\lambda p$ are workers in the automobile industry. Those with $q \in (\hat{q}, q_a)$ work in the software industry, while those with abilities $q \in (q_a, q_b)$ are managers of automobile firms.

Define q_w so that $\hat{F}(q_w) - w = w$, where $\hat{F}(z) \equiv \frac{1}{\Phi(\hat{q})} \int_{q_{\min}}^{\hat{q}} F(z, q) d\Phi(q)$ is the expected output of an automobile firm with a manager of talent z when the worker pool includes all individuals with talent between $q_{\min}$ and $\hat{q}$. With this definition, q_w is at the intersection of MM and WW; therefore $q_w > \hat{q}$. Also, the MM curve is steeper at q_w than it is at q_a, and it is steeper at q_a than is the EE curve. Therefore, $\hat{F}'(q_w) > \lambda p$.

But $q_w > \hat{q}$ implies $\hat{F}(q_w)/2q_w < \lambda p$, by the definitions of $q_w$and $\hat{q}$. Also, the symmetry of $F(\cdot)$ and the fact that it has an elasticity of substitution everywhere less than one implies $q_w F_1(q_w, q)/F(q_w, q) \leq 1/2$ for all $q \leq q_w$. This in turn implies

$$\hat{F}'(q_w) = \frac{1}{\Phi(\hat{q})} \int_{q_{\min}}^{\hat{q}} F_1(q_w, q) d\Phi(q) < \frac{1}{\Phi(\hat{q})} \int_{q_{\min}}^{\hat{q}} \frac{F(q_w, q)}{2q_w} d\Phi(q) d\Phi(q) = \frac{\hat{F}(q_w)}{2q_w}.$$

Again, we have a contradiction. So there can be no equilibrium of the sort depicted in Figure 7.

Appendix B

This appendix derives the comparative statics of the model under the assumption of a uniform distribution of talents. In the process, I substantiate the various claims made in the text.

Let $q \sim U[q_{\min}, q_{\max}]$, where $q_{\min} = \bar{q} - e$, $q_{\max} = \bar{q} + e$, and $\bar{q}$ is the mean of q. Combining (1) and (2), and using the properties of the uniform distribution, we have

$$\lambda p q_m = \frac{1}{q_w - q_{\min}} \int_{q_{\min}}^{q_w} \left[F(q_m, q) - \frac{F(q_w, q)}{2} \right] dq \ . \tag{A1}$$

With q uniformly distributed, (3) becomes

$$q_m = 2q_w - q_{\min} \ . \tag{A2}$$

Differentiating (A1) and (A2) totally with respect to p gives

$$\frac{dq_m}{dp} = \frac{1}{\Delta} \left(\frac{\lambda q_m}{e} \right) \tag{A3}$$

where

$$\begin{aligned} \Delta \equiv & -2\lambda p + \\ & \frac{1}{q_w - q_{\min}} \left\{ \int_{q_{\min}}^{q_w} \left[2F_1(q_m, q) - \frac{F_1(q_w, q)}{2} \right] dq - \tilde{F}(q_m) + \frac{\tilde{F}(q_w)}{2} + F(q_m, q_w) - f q_w \right\} \end{aligned} \tag{A4}$$

and $\tilde{F}(z) = \frac{1}{q_w - q_{\min}} \int_{q_{\min}}^{q_w} F(z, q) dq$ is the expected output for a manager of ability z.

First I will establish that $\Delta < 0$. To this end, I use (A1) to solve for λp, and substitute the result in (A4), to derive[17]

$$\begin{aligned} \frac{q_m(q_w - q_{\min})}{q_{\min}} \Delta &= -\tilde{F}(q_m) + \frac{3q_w - 2q_{\min}}{2q_w} \tilde{F}(q_w) - F(q_m, q_w) + 2F(q_m, q_{\min}) - \frac{q_m}{2q_w} F(q_w, q_{\min}) \\ &= \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + \Delta_5 \ , \end{aligned}$$

[17] Note that Euler's theorem and integration by parts gives

$$\int_{q_{\min}}^{q_w} F_1(z, q) dq = \frac{1}{z} \left[2(q_w - q_{\min}) \tilde{F}(z) - q_w F(z, q_w) + q_{\min} F(z, q_{\min}) \right] .$$

where

$$\Delta_1 \equiv F(q_m, q_{\min}) + F(q_w, q_w) - F(q_m, q_w) - F(q_w, q_{\min}) \ ,$$

$$\Delta_2 \equiv \tilde{F}(q_w) + F(q_m, q_{\min}) - \tilde{F}(q_m) - F(q_w, q_{\min}) \ ,$$

$$\Delta_3 \equiv \frac{1}{2}\left[\tilde{F}(q_w) - F(q_w, q_w)\right] \ ,$$

$$\Delta_4 \equiv \frac{q_{\min}}{2q_w}\left[F(q_w, q_{\min}) + F(q_w, q_w) - 2\tilde{F}(q_w)\right] \ ,$$

$$\Delta_5 \equiv \frac{1}{2}\left[2F(q_w, q_{\min}) - F(q_w, q_w) - F(q_{\min}, q_{\min})\right] \ .$$

Now, Δ_1, Δ_2, and Δ_5 are negative by the supermodularity of $F(\cdot)$, Δ_3 is negative because $q_w \geq q$ for all $q \in [q_{\min}, q_w]$, and Δ_4 is negative because $F_{12}(q_w, q) > 0$ and $F(\cdot)$ homogeneous of degree one implies $F_{22}(q_w, q) < 0$ for $q \in [q_{\min}, q_w]$. Thus, $\Delta < 0$.

With $\Delta < 0$, (A3) implies $dq_m/dp < 0$. But the output of software is inversely related to q_m; $x_s = \frac{\lambda L}{2e}\int_{q_m}^{q_{\max}} q dq$. Therefore, $dx_s/dp > 0$.

Also, $dq_w/dp = (dq_m/dp)/2$. Therefore, $dq_w/dp < 0$. From (1) and the properties of the uniform distribution, we have

$$w = \frac{\tilde{F}(q_w)}{2}.$$

Then $\partial w/\partial q_w > 0$, which means that $dw/dp < 0$.

Now I differentiate (A1) and (A2) with respect to e, holding p constant. After some rearranging, this yields

$$\begin{aligned}\frac{dq_m}{de} \quad &= \quad \frac{1}{(q_w - q_{\min})\Delta} \times \\ &\left[\tilde{F}(q_m) - \frac{\tilde{F}(q_w)}{2} - F(q_m, q_{\min}) - \frac{F(q_w - q_{\min})}{2} + F(q_m, q_w) - f q_w - \int_{q_{\min}}^{q_w} \frac{F_1(q_w, q)}{2} dq\right] \ .\end{aligned}$$

Using the definition of Δ, the expression for λp from (A1) and the relationship between q_m and q_w from (A2), I calculate

$$\frac{dq_m}{de} + 1 = \frac{-2}{\Delta}\left\{\frac{2F(q_m, q_{\min})}{q_m} - \frac{[F(q_m, q_w) + \tilde{F}(q_m)]}{q_m} - \frac{F(q_w, q_{\min})}{2q_w} + \left[\frac{1}{q_w} - \frac{1}{2q_m}\right]\tilde{F}(q_w)\right\}$$

or

$$\frac{dq_m}{de} + 1 = \frac{-2}{\Delta}\left\{\frac{q_w - q_{\min}}{q_{\min}}\Delta - \frac{1}{2q_w}\left[F(q_m, q_{\min}) - F(q_w, q_{\min})\right]\right\} \ . \qquad \text{(A5)}$$

Note that $-2/\Delta > 0$, while $\Delta < 0$ and $q_m > q_w$, which implies that the term in curly brackets in (A5) is negative. Therefore, $dq_m/de + 1 < 0$ and, *a fortiori*, $dq_m/de < 0$.

From $x_s = \frac{\lambda L}{2e}\int_{q_m}^{q_{\max}} q dq = \lambda L[(\bar{q} + e)^2 - q_m^2]/4e$, I compute

$$\frac{dx_s}{de} = \frac{\lambda L}{4e^2}\left[q_m^2 + e^2 - \bar{q}^2 - 2eq_m\frac{dq_m}{de}\right] \ .$$

But $q_m > q_{\min} = \bar{q} - e$ implies $eq_m > e\bar{q} - e^2$ and $q_m^2 > \bar{q}^2 - 2\bar{q}e + e^2$. Therefore,

$$\frac{dx_s}{de} > \frac{\lambda L}{4e^2}\left[2e^2 - 2\bar{q}e - 2e(\bar{q}-e)\frac{dq_m}{de}\right] = -\lambda L\frac{\bar{q}-e}{2e}\left[\frac{dq_m}{de}+1\right] .$$

So $dq_m/de + 1 < 0$ implies $dx_s/de > 0$; i.e., a spread in the distribution of talent increases equilibrium output of software.

Output of automobiles is given by

$$x_a = \frac{L}{2e(q_w - q_{\min})}\int_{q_w}^{q_m}\int_{q_{\min}}^{q_w} F(z,q)dqdz .$$

From this I compute

$$\begin{aligned}\frac{dx_a}{de} \quad = \quad & -\frac{x_a}{e(q_w - q_{\min})}\left[e + q_w - q_{\min}\right] + \\ & \frac{L}{2e(q_w - q_{\min})}\left\{\left[\int_{q_w}^{q_m} F(z,q_w)dz\right]\frac{dq_w}{de} + \int_{q_w}^{q_m} F(z,q_{\min})dz\right\} + \\ & \frac{L}{2e}\left[\tilde{F}(q_m)\frac{dq_m}{de} - \tilde{F}(q_w)\frac{dq_w}{de}\right] \end{aligned} \tag{A6}$$

Note from (A2) and $q_m = \bar{q} - e$ that

$$\frac{dq_w}{de} = \frac{1}{2}\frac{dq_m}{de} - \frac{1}{2} .$$

With $dq_m/de < -1$, this implies $dq_m/de < dq_w/de < -1$.

The term on the first line on the right-hand side of (A6) is negative, because $q_w > q_{\min}$. The term on the second line of (A6) is negative, because $F(z,q_w) > F(z,q_{\min})$ and $dq_w/de < -1$. The term on the third line of (A6) is negative, because $\tilde{F}(q_m) > \tilde{F}(q_w)$ and $dq_m/de < dq_w/de < 0$. I conclude that $dx_a/de < 0$; i.e., a spread in the distribution of talent reduces equilibrium output of automobiles.

Since $dx_s/de > 0$ and $dx_a/de < 0$, $d(x_s/x_a)/de > 0$. That is, a spread in the distribution of talent increases relative output of software at a given price. It follows that a country with a more diverse labor force produces relatively more software in a free-trade equilibrium. With identical and homothetic preferences, this country must export software and import automobiles.